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Modeling of fluid-solid interfaces by the Discrete Wave Number

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Abstract

This work shows the wave propagation in fluid-solid interfaces due to dynamic excitations. The interface connects an acoustic medium (fluid) and a solid one, a wide range of elastic solid materials is considered. By means of an analysis of diffracted waves in a fluid, it is possible to deduce the mechanical characteristics of the solid medium, specifically, its wave propagation velocity. For this purpose, the Discrete Wave Number method (DWN) is formulated to deal with this problem. This method usually models ground motions, where the wave radiated from a source is expressed as the wave number integration. The validation was performed by means of results comparison with published research determined by Boundary Elements Method. Firstly, spectra of pressures for each solid material considered are displayed. Then, the Fast Fourier Transform algorithm to obtain results in the time domain is applied, where the emergence of Scholte's interface waves and the amount of energy that they carry are evinced.

Key words: interface waves, Scholte's waves, elastic waves, fluid-solid interface, Discrete Wave Number method (DWN)

1. Introduction

The study of interface waves that propagate in the vicinity of a fluid medium that interacts with an elastic solid medium has its origins in the pioneering work by Scholte in 1942 and 1947 [1, 2] and therefore, this kind of waves is known as Scholte's waves. These waves belong to one of the three basic types of interface waves presented in isotropic media. The other two types of interface waves are the Rayleigh and Stoneley's waves, for interfaces between vacuum-solid and solid-solid media, respectively [3, 4].

In the interface waves, the major part of the energy is located in the interface and decreases exponentially as a function of depth. However, interface waves energy decays less rapidly with the distance than the compression and shear waves [5]. This concentration of energy has enormous implications in some areas of physics and engineering. For example, the Rayleigh's waves are studied extensively in the earthquake engineering and seismology due to their catastrophic results during strong earthquakes.

Other applications for particular cases, focusing mainly on the phenomenon of interface waves in seabed, highlighting some specific characteristics on wave propagation in interfaces, such as attenuation, porosity, etc. have been divulged in [6–14].

In the field of numerical methods for the study of this phenomenon, there are several formulations designed for modeling complex configurations of interfaces. Some of these methods include: Finite Element [15], Finite Differences [16, 17], Boundary Element [18, 19], Spectral and Pseudo spectral [20–22], among others.

In this article, we established the use of the Discrete Wave Number method (DWN) for studying fluid-solid interfaces, for a wide range of solid materials. In this method the wave that is radiated from

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a source is expressed as the integration in the wave number domain and by superposition of plane waves.

The following section summarizes the main equations used to develop the DWN. The results obtained can be used to determine the wave propagation velocities in solids by measurement of diffracted fields of pressures in the fluid.

2. Formulation by means of the Discrete Wave Number method (DWN)

DWN is one of the techniques used to simulate wave propagation. The wave that is radiated from a source is expressed as the integration in the wave number domain [23]. The main idea of the method is to represent the source as a superposition of plane waves that propagate in discrete angles. Because the media lack of inelastic damping, the denominator of the integrating becomes zero for a particular wave number and therefore, the numerical integration becomes undefined. To solve this problem, a complex frequency is incorporated in the formulation [23].

The source representation is made by the superposing of plane waves in the wave number domain as it is established by the Fourier transform theory in multiple dimensions. The discretization of the angles is done by the fact that the angle in which each plane wave is propagating is given by the wave number vector, and the Fourier integral is realized over all the wave number space.

Pulse incident in the fluid (source), as shown in Fig. 1a, can be expressed as:

$$\begin{aligned} p^{0^{\mathrm{F}}}(\boldsymbol{x}) &= C(\omega) H_0^{(2)}\left(\omega r / c^{\mathrm{F}}\right) = \\ &= \frac{C(\omega)}{\pi} \int_{-\infty}^{\infty} \frac{\mathrm{e}^{-\mathrm{i}kx_1 - \mathrm{i}\eta|x_3|}}{\eta} \mathrm{d}k \approx \\ &\approx \frac{C(\omega)}{\pi} \sum_{n=-N}^{N} \frac{\mathrm{e}^{-\mathrm{i}k_n x_1 - \mathrm{i}\eta_n|x_3|}}{\eta_n} \Delta k, \end{aligned} \tag{1}$$

where $p^{0^{\mathrm{F}}}(\boldsymbol{x})$ is incident pulse at the fluid, $\boldsymbol{x} = \{x_1, x_3\}$, $C(\omega)$ is scale factor for the incident pulse, $H_0^{(2)}(\bullet)$ is Hankel function of second kind and zero order, ω is circular frequency, c^{F} is compressional wave velocity in the fluid and $r = r(\boldsymbol{x})$ is the distance from the receiver to the source. k is wave number. $\eta = \sqrt{\frac{\omega^2}{c^{\mathrm{F}^2}} - k^2}$ with $\mathrm{Im}\ \eta < 0$. If we express k in discrete values, then, $k_n = n\Delta k$ and $\eta_n = \sqrt{\frac{\omega^2}{c^{\mathrm{F}^2}} - k_n^2}$ with $\mathrm{Im}\ \eta_n < 0$.

If we assume that the whole pressure and displacement fields in the fluid are represented as the sum of an incident and the diffracted fields, these may be expressed, respectively, as:

$$\begin{aligned} p^{\mathrm{F}}(\boldsymbol{x}) &= p^{0^{\mathrm{F}}}(\boldsymbol{x}) + p^{d^{\mathrm{F}}}(\boldsymbol{x}) = \\ &= p^{0^{\mathrm{F}}}(\boldsymbol{x}) + \sum_{n=-N}^{N} A_n\, \mathrm{e}^{-\mathrm{i}k_n x_1 + \mathrm{i}\eta_n(x_3 - a)} \end{aligned} \tag{2}$$

and

$$\begin{aligned} u_3^{\mathrm{F}}(\boldsymbol{x}) &= \frac{1}{\rho\omega^2} \frac{\partial p^{\mathrm{F}}(\boldsymbol{x})}{\partial x_3} = \\ &= \frac{1}{\rho\omega^2} \left\{ \sum_{n=-N}^{N} \frac{-\mathrm{i}\,\mathrm{sign}(x_3)}{\pi} \mathrm{e}^{-\mathrm{i}k_n x_1 - \mathrm{i}\eta_n|x_3|} \Delta k + \right. \\ &\left. + \sum_{n=-N}^{N} \mathrm{i} A_n \eta_n \mathrm{e}^{-\mathrm{i}k_n x_1 + \mathrm{i}\eta_n|x_3 - a|} \right\}. \end{aligned} \tag{3}$$

For a solid, we consider that the displacement potentials have the form $\phi = \sum B_n \mathrm{e}^{-\mathrm{i}k_n x_1} \mathrm{e}^{-\mathrm{i}\gamma_n(x_3 - a)}$ and $\psi = \sum C_n \mathrm{e}^{-\mathrm{i}k_n x_1} \mathrm{e}^{-\mathrm{i}\upsilon_n(x_3 - a)}$, where $\gamma_n = \sqrt{\frac{\omega^2}{\alpha^2} - k_n^2}$ with $\mathrm{Im}\ \gamma_n < 0$, and $\upsilon_n = \sqrt{\frac{\omega^2}{\beta^2} - k_n^2}$ with $\mathrm{Im}\ \upsilon_n < 0$. α and β are the compressional and shear wave velocities for the solid, respectively; a is distance between the source and the solid boundary.

The displacement field for the solid can be expressed as: $u = \frac{\partial\phi}{\partial x_1} - \frac{\partial\psi}{\partial x_3}$ and $w = \frac{\partial\phi}{\partial x_3} + \frac{\partial\psi}{\partial x_1}$. The stress field is derived using the well-known equation:

$$\sigma_{ij}(\boldsymbol{x}) = \lambda\varepsilon_{kk}\delta_{ij} + 2\mu\varepsilon_{ij}, \tag{4}$$

where $\sigma_{ij}(\boldsymbol{x})$ is stress tensor, λ, μ are the Lamé's constants, ε_{ij} is strain tensor and δ_{ij} is Kronecker's delta.

Boundary conditions are represented by the following equations:

$$u_3^{\mathrm{S}}(\boldsymbol{x}) = u_3^{\mathrm{F}}(\boldsymbol{x}), \quad \forall \boldsymbol{x} \in \partial S = \partial F, \tag{5}$$

$$t_1^{\mathrm{S}}(\boldsymbol{x}) = 0, \quad \forall \boldsymbol{x} \in \partial S, \tag{6}$$

$$t_3^{\mathrm{S}}(\boldsymbol{x}) = -p^{\mathrm{F}}(\boldsymbol{x}), \quad \forall \boldsymbol{x} \in \partial S. \tag{7}$$

Once the boundary conditions were applied, the unknown coefficients A_n, B_n and C_n are obtained and the whole pressure field in the fluid is then finally calculated by means of Eq. (2). The system of equations to be solved is given by:

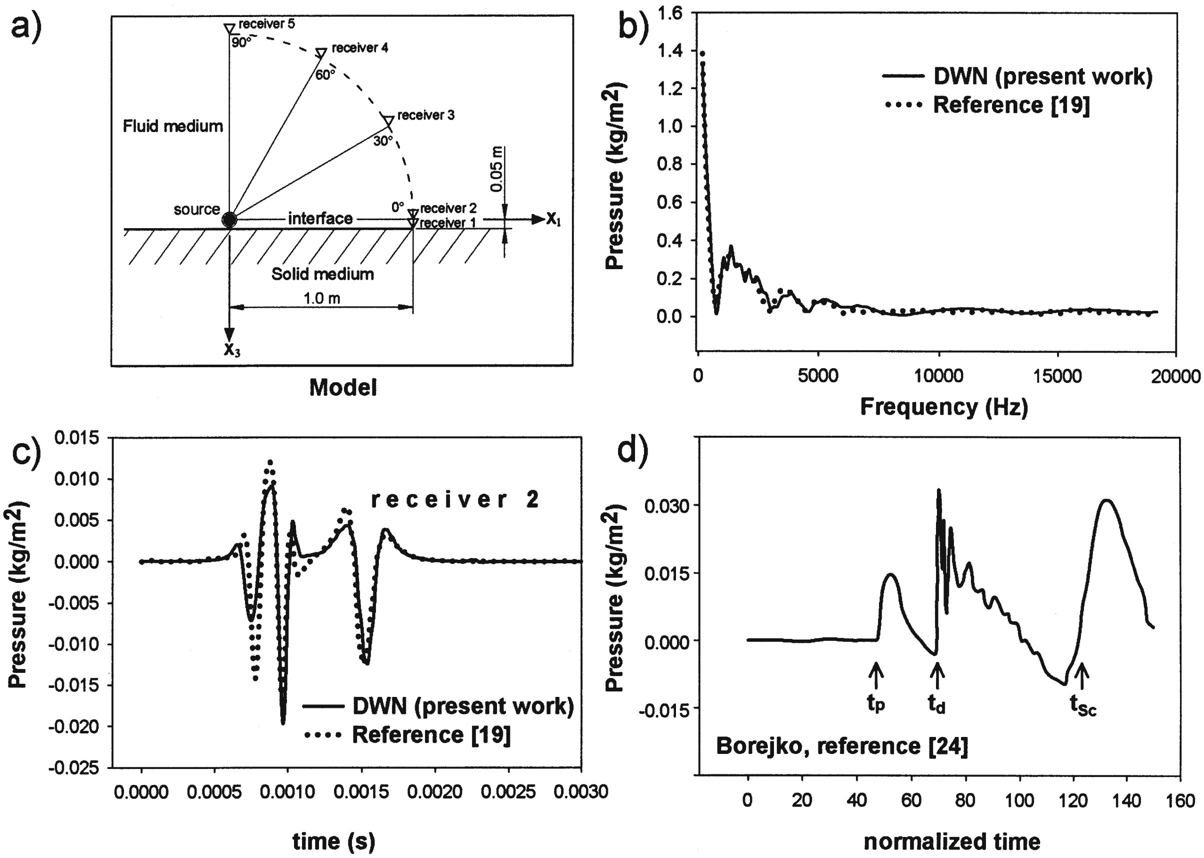

Fig. 1. a) Fluid-solid interface to be solved by means of DWN; b) and c) spectra and synthetic seismograms of pressures, respectively, registered by the receiver 2. The results obtained by DWN are displayed with a solid line, while those obtained by the Boundary Element Method [19] are drawn with a dotted line; d) experimental results obtained by Borejko [24] in the case of a triangular source. The agreement and the consistency of the results are very satisfactory.

$$\begin{bmatrix} \dfrac{-\mathrm{i}\eta_n}{\rho\omega^2} & -\mathrm{i}\gamma_n & -\mathrm{i}k_n \\ 1 & -\lambda(k_n^2+\gamma_n^2)-2\mu\gamma_n^2 & 2\mu k_n \upsilon_n \\ 0 & -2k_n\gamma_n & \nu_n^2-k_n^2 \end{bmatrix} \begin{bmatrix} A_n \\ B_n \\ C_n \end{bmatrix} = \\ = \begin{bmatrix} -\dfrac{\mathrm{i}\Delta k}{\rho\omega^2\pi}\mathrm{e}^{-\mathrm{i}\eta_n a} \\ -\dfrac{\Delta k}{4\pi\eta_n}\mathrm{e}^{-\mathrm{i}\eta_n a} \\ 0 \end{bmatrix}. \quad (8)$$

3. Validation

To show the accuracy of our formulation, we take the case of a Water-Pitch interface, which was also considered by Borejko [24] and Rodríguez-Castellanos et al. [19]. The material properties considered are: compressional wave velocity of 2443 m s^{-1}, shear wave velocity of 1000 m s^{-1} and a density of 1270 kg m^{-3}, for the Pitch. The compressional wave velocity and water density are 1501 m s^{-1} and 1000 kg m^{-3}, respectively. Borejko [24] considered a triangular pulse as a source in the fluid, through theoretical and experimental studies showing the emergence of interface waves. For the validation of DWN we take a Ricker pulse, which was also considered in Ref. [19] using a Boundary Element Formulation, to verify the accuracy of our formulation.

Figure 1a shows the interface and the receiver array where pressures are measured. For all these cases, the initial pressure (source) is generated at a distance of 0.05 m from the elastic solid boundary. Five receivers are placed at a distance of 1.0 m from the source. Receiver 2 is placed at a horizontal distance of 1.0 m from the source (as shown in Fig. 1a) in order to validate the results obtained by the DWN with respect to those by a Boundary Element Formulation [19]. In Fig. 1b the results of DWN and Ref. [19] are plotted. It shows the spectrum of pressure measured in receiver 2 for a frequency range from 0 to 19200 Hz. The results obtained by the DWN are plotted with a solid line, while those obtained from Ref. [19] are plotted with a dotted

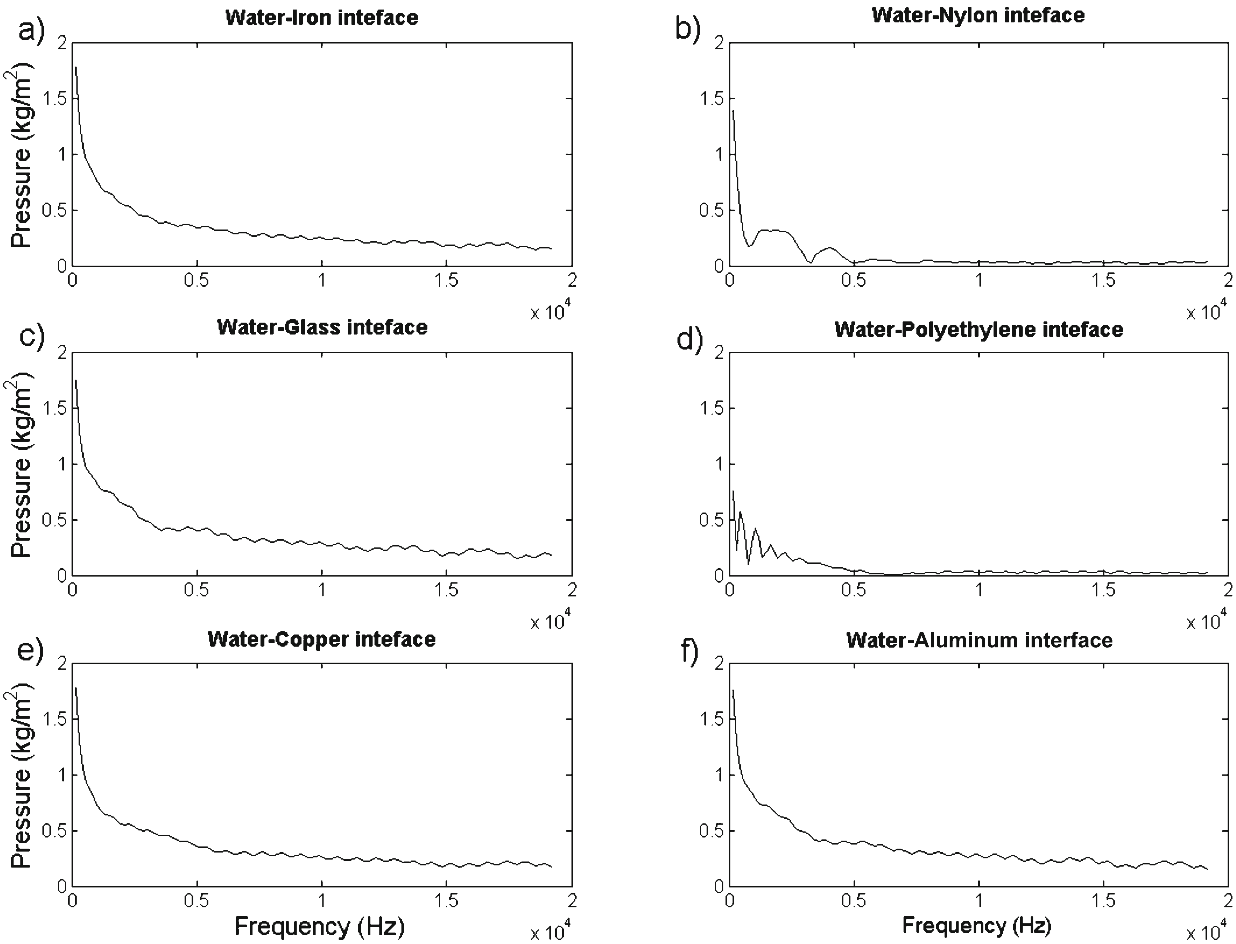

Fig. 2a–f. Spectra of pressures for the interface cases shown in Table 1 are displayed. For these cases, the receiver is located at a distance of 0.05 m from solid boundary and a horizontal distance from the source of 1.0 m (Fig. 1a, receiver 2).

line, good agreement is observed. There can be seen slight resonance peaks recorded in receiver 2. Figures 1c,d show synthetic seismograms of pressures. From Fig. 1c results obtained through the present formulation and those obtained in Ref. [19] are validated, once more good agreement between the two formulations can be seen. Figure 1d shows results by Borejko [24], which correspond to a triangular pulse. In the latter two figures, arrival times of different wave fronts are identified and they can easily be associated with the propagation velocities of the two materials at the interface. The wave fronts measured, t_{p}, t_{d} and t_{Sc} have velocities of 2443 $\mathrm{m\,s^{-1}}$, 1501 $\mathrm{m\,s^{-1}}$ and 823.5 $\mathrm{m\,s^{-1}}$, respectively. They can be linked with the compressional wave velocity by Pitch, the direct wave velocity for the water and Scholte's interface waves, respectively. These speeds agree satisfactorily for the three methods involved, including the present work.

4. Numerical examples

To develop several numerical examples of Fluid--Solid interfaces we take a wide range of solid material properties [25], characterized by its wave propagation velocities and densities. The properties of six materials that were used in the calculations are shown in Table 1.

Table 1. Properties of materials used for numerical examples

Model	α ($\mathrm{m\,s^{-1}}$)	β ($\mathrm{m\,s^{-1}}$)	ρ ($\mathrm{kg\,m^{-3}}$)
Water-Aluminum	6420	3040	2700
Water-Cooper	5010	2270	8930
Water-Glass	5640	3280	2240
Water-Iron	5900	3200	7690
Water-Nylon	2600	1100	1120
Water-Polyethylene	1950	540	900
Water, for all cases	1501	–	1000

Figure 2 shows the pressure spectra for the six materials analyzed. For all these cases, the initial pressure (source) was generated at a distance of 0.05 m from the elastic solid boundary and registered at a

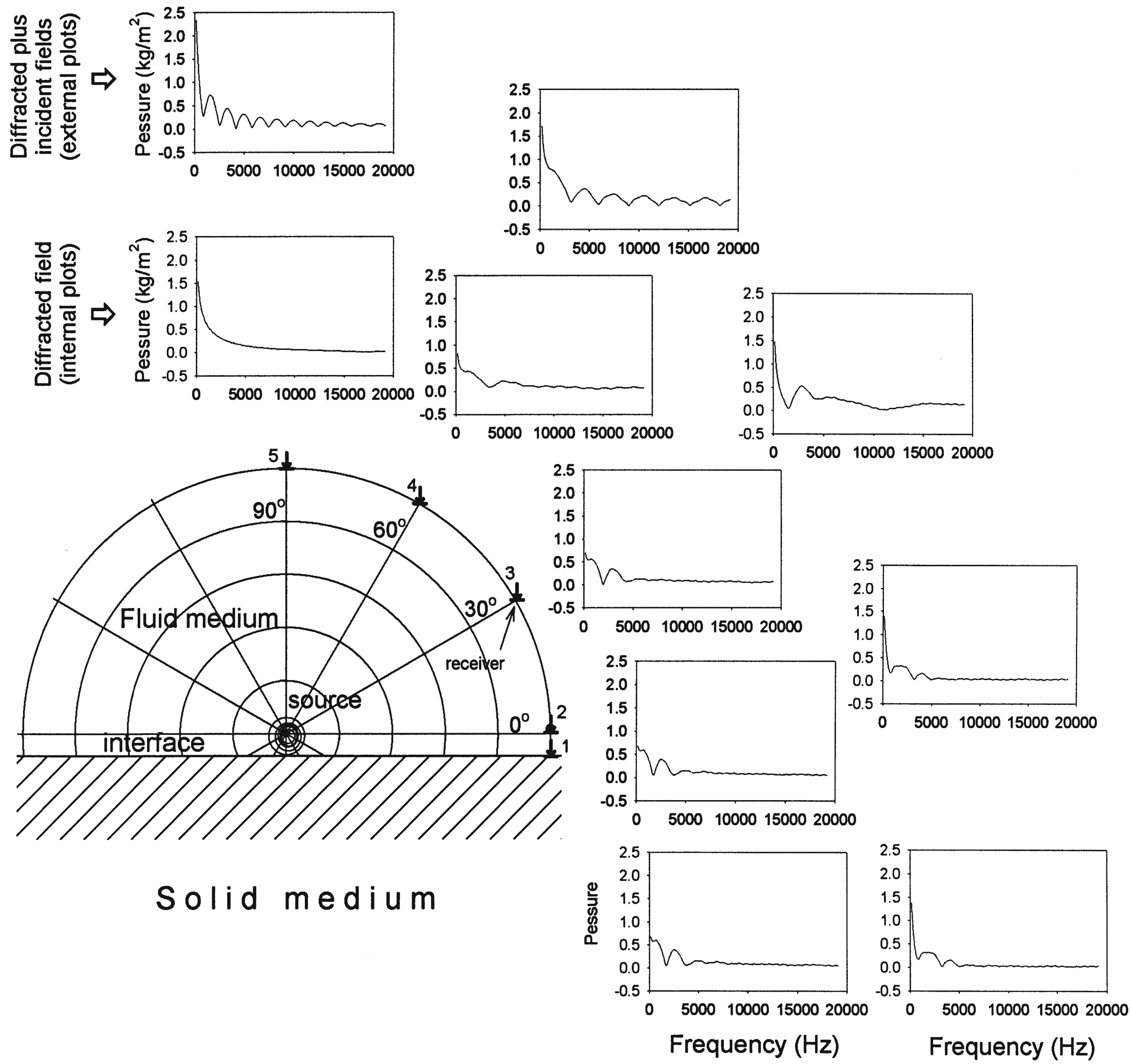

Fig. 3. Spectra of pressures for the Water-Nylon interface registered in five receivers. Diffracted fields are the internal plots and total fields are the external plots for each receiver.

horizontal distance of 1.0 m from the source (as seen in Fig. 1a, receiver 2). The frequency analysis is done considering frequency increments of 150 Hz to reach a maximum of 19200 Hz.

In this figure, results obtained by the DWN are exhibited. One can also see that resonance effects are slightly manifested for models of Water-Nylon and Water-Polyethylene (Fig. 2b,d). However, in all cases, the curves describe a very stable behavior and tend to be asymptotic from frequency 6000 Hz.

For the rest of the models (Fig. 2a,c,e,f) the pressure spectra show a monotonous behavior describing small oscillations in each case.

Figure 3 shows the pressure field for the Water-Nylon interface for the five receivers shown in Fig. 1a. This model was analyzed using an increment of frequency 150 Hz, reaching a final frequency of 19200 Hz. In this figure, the diffracted fields (internal plots) and the total fields (external plots) measured in each receiver are drawn. The main variations of the diffracted field are registered by receivers 1 to 3, this is due to its proximity with the interface, while for receivers 4 and 5 the variations of the diffracted field tend to be slight. However, the total field variation for receivers 1 to 3 becomes practically negligible from 6000 Hz, while for receivers 4 and 5 variations are clearly evident.

Figure 4 shows pressure variations in the time domain, expressed through synthetic seismograms from

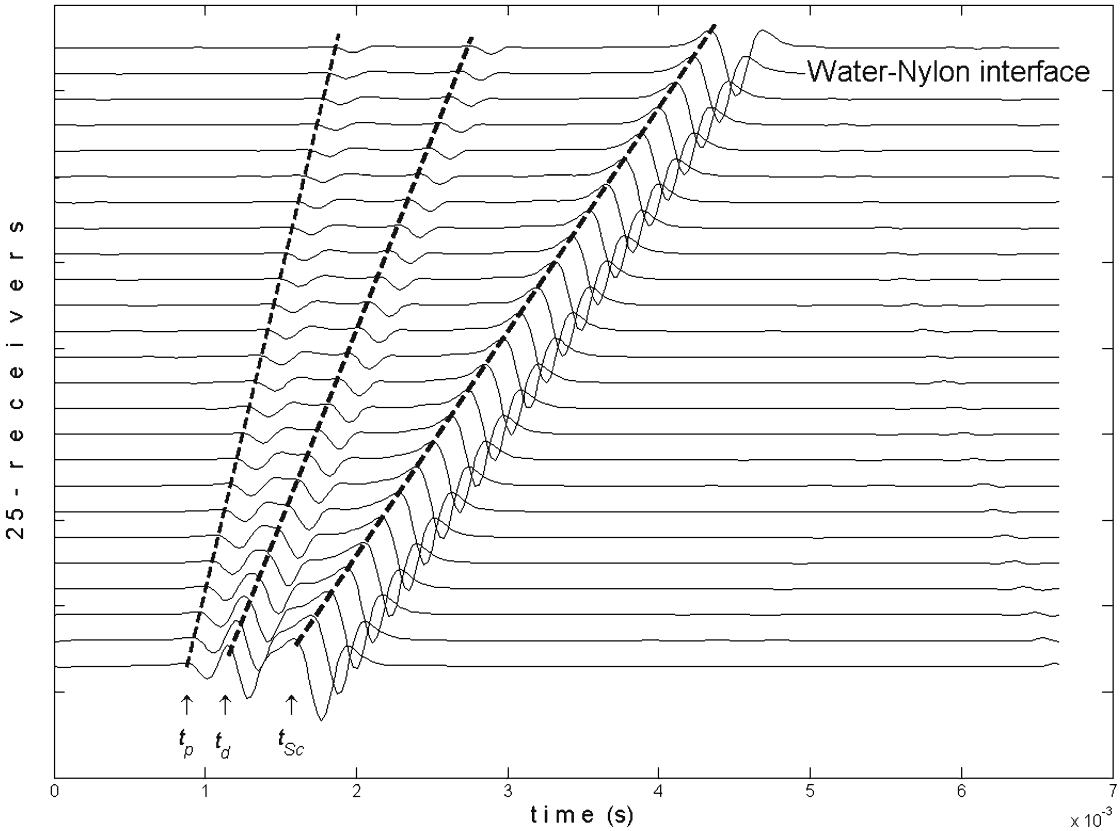

Fig. 4. Synthetic seismograms of pressures obtained by means of the DWN for the case of Water-Nylon interface, registered by 25 receivers.

25 receivers. For this purpose, the Fast Fourier Transform (FFT) and a pulse temporary Ricker were used. By means of the FFT it is possible to observe the different wave types that emerge in these interfaces. The first receiver was placed at a distance of 1.0 m from the source, and the rest of the receivers were located at a distance increment of 0.05 m. The distance between the source and the receivers at the elastic solid was 0.05 m.

It is also clear that through the use of synthetic seismograms it is possible to observe the influence of the Nylon compressional wave velocity, α, also known as P wave velocity, represented as t_{p}. The direct wave that travels in the fluid and is sensed by the receiver is labeled as t_{d} and Scholte's interface wave as t_{Sc}. In this case, the velocities measured were $t_{\mathrm{p}} = 2600\,\mathrm{m\,s^{-1}}$, $t_{\mathrm{d}} = 1500\,\mathrm{m\,s^{-1}}$ and $t_{\mathrm{Sc}} = 937.5\,\mathrm{m\,s^{-1}}$, the first two values are consistent with the velocities shown in Table 1, the last of them corresponds to Scholte's interface waves, which carry the most of the energy.

5. Conclusion

In this paper we have formulated the Discrete Wave Number method to study the propagation of elastic waves in fluid-solid interfaces. In this technique, the wave that is radiated from a source is expressed as an integration in the wave number domain, where the main idea of the method is to represent the source as a superposition of plane waves that propagate in discrete angles.

The analysis was performed using a wide range of solid materials characterized by its wave velocities and densities. In the spectra shown, it is possible to observe that the cases for Water-Nylon and Water-Polyethylene show slight resonance effects. However, in all cases, from 6000 Hz frequency the pressure spectra show an asymptotic behavior, describing minor oscillations.

In time domain, we have concluded that by means of an analysis of diffracted waves in a fluid, it is possible to deduce the mechanical characteristics of the solid medium, specifically, its wave propagation velocities.

The results obtained by our technique were compared with those obtained by the Boundary Element Method and experimental developments by Borejko. In all cases, good agreement between the methods is confirmed. The existence and propagation of Scholte's waves is evidently observed, where the most of the energy is carried.

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